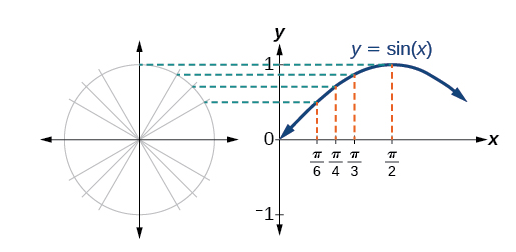
# Graphing Sine and Cosine Functions

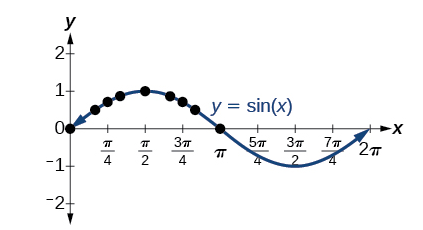
Recall that the sine and cosine functions relate real number values to the and coordinates of a point on the unit circle. We can graph these functions by using a table of values. The values for the sine function are shown in the table.

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

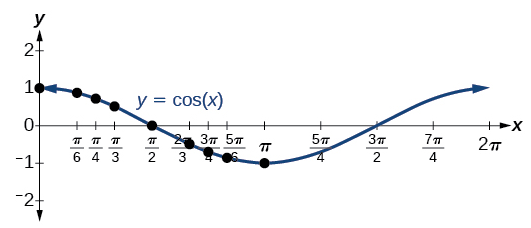
Plotting these points, we can see that the sine function is a curve, where each of the sine values corresponds to the values of the sine function (i.e. coordinates) in quadrants I and II on the unit circle.



If we include the negative sine values (which correspond to the values of the sine function in quadrants III and IV on the unit circle), we can see that the function makes a “wave”.

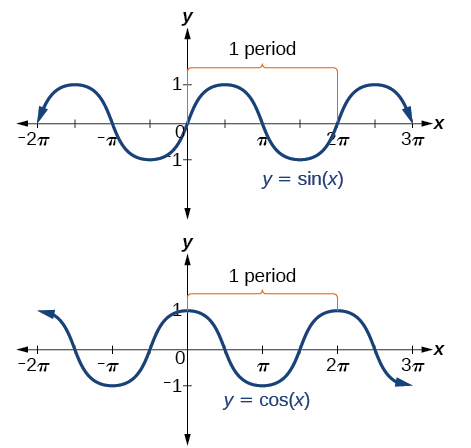


The cosine function can be graphed in a similar way.



Because we can evaluate the sine and cosine of any real number, both of these functions are defined for all real numbers and the graphs will just repeat the same shape.

In both sine and cosine graphs, the shape of the graph repeats after , which means the functions are periodic with a period of .



## Characteristics of Sine and Cosine Functions

The sine and cosine functions have several distinct characteristics.

• They are periodic functions with a period of .

• The domain of each function is and the range is .

• The graph of is symmetric about the origin because it is an odd function.

• The graph of is symmetric about the -axis because it is an even function.

Examples:

1. How does the graph of compare with the graph of ? Explain how you could horizontally translate the graph of to obtain .
2. Why are the sine and cosine functions called periodic functions?

# Investigating Sinusoidal Functions

Sine and cosine functions have a regular period and range. If we watch ocean waves or ripples on a pond, we will see that they resemble sine and cosine functions. However, they are not necessarily identical, as some are taller or longer than others.

A **sinusoidal function** has the same general shape as a sine or cosine function. The general forms of sinusoidal functions are

and

## Determining the Period of Sinusoidal Functions

Sinusoidal functions are essentially transformations of sine and cosine functions. Using what we know about transformations (more specifically horizontal stretches or shrinks), we can determine the period.

Using the general form equations of the sinusoidal functions

and ,

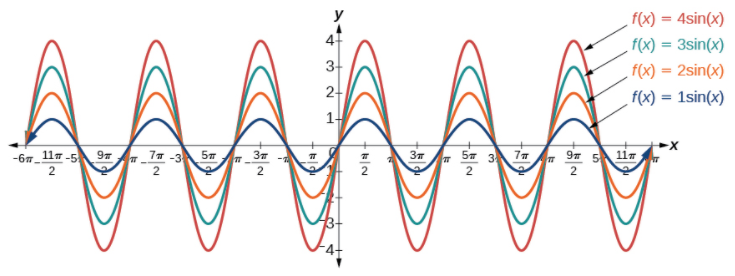
the **period** is .

Example

Determine the period of the function .

## Determining Amplitude

We just saw that the variable relates to the period of a function. The variable is related to the **amplitude**, or greatest distance from rest (the **midline**, which is the -axis unless there is a vertical shift). represents the vertical stretch factor, and its absolute value is the amplitude.



Using the general form equations of the sinusoidal functions

and

the **amplitude** is , which is the vertical height from the **midline**. In addition,

Example

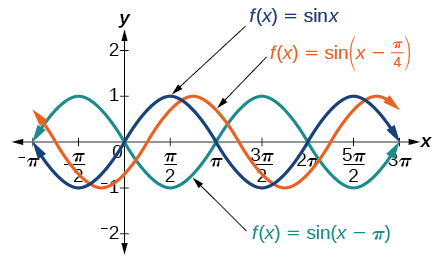
What is the amplitude of the sinusoidal function ? Is the function stretched or compressed vertically?

# Analyzing Graphs of Variations of and

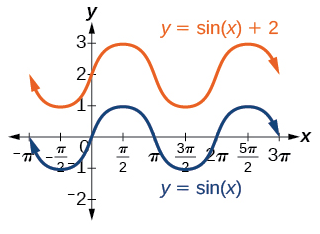
Now that we understand how and relate to the general form equation for the sine and cosine functions, we will explore the variables and .

Given the equation in the form or , is the **phase shift** (the horizontal displacement of the basic sine and cosine function) and is the **vertical shift**.

If , the graph shifts to the right. If , the graph shifts to the left. An example is shown below.



If , the graph shifts up. If , the graph shifts down.



Example

Determine the phase shift for .

Given a sinusoidal function in the form , identify the midline, amplitude, period, and phase shift.

Determine the amplitude as .

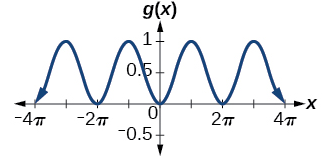
Determine the period as .

Determine the phase shift as .

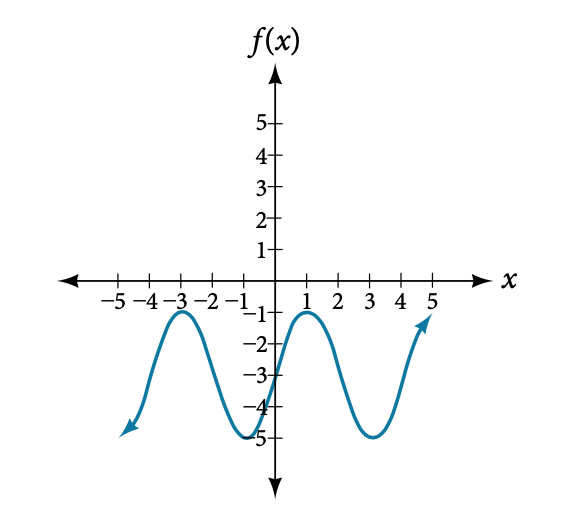
Determine the midline as .

Examples

1. Determine the midline, amplitude, period, and phase shift of the function .
2. Determine the midline, amplitude, period, and phase shift of the function .
3. Determine the formula for the cosine function in the figure below.



1. Determine the amplitude, period, midline, and a formula for the function in the figure below.



# Graphing Variations of and

To begin graphing variations of sine and cosine functions, we will begin with a simplified form. Then we will add in horizontal and vertical shifts.

Given the function , sketch its graph.

Identify the amplitude, .

Identify the period, .

Start at the origin, with the function increasing to the right if is positive or decreasing if is negative.

At there is a local maximum for or a minimum , with .

The curve returns to the -axis at .

There is a local minimum for (maximum for ) at with .

The curve returns again to the -axis at .

The cosine function will behave similarly but begin at the maximum if is positive and at the minimum if is negative.

Examples

1. Sketch a graph of .
2. Sketch a graph of . Determine the midline, amplitude, period, and phase shift.

Given a sinusoidal function with a phase shift and a vertical shift, sketch its graph.

Express the function in general form or .

Identify the amplitude .

Identify the period .

Identify the phase shift .

Draw the graph of or shifted to the right or left by and up or down by .

Examples

1. Sketch a graph of .
2. Determine the midline, amplitude, period, and phase shift of . Then graph the function.

# Using Transformations of Sine and Cosine Functions

Examples

1. A Ferris wheel is 25 meters in diameter and boarded from a platform that is 1 meter above the ground. The six o’clock position on the Ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 10 minutes. The function gives a person’s height in meters above the ground minutes after the wheel begins to turn.
   1. Find the amplitude, midline, and period of .
   2. Find a formula for the height function .
   3. How high off the ground is a person after 5 minutes?
2. A weight is attached to a spring that is then hung from a board, as shown in the figure below. As the spring oscillates up and down, the position of the weight relative to the board ranges from inches (at time ) to inches (at time ) below the board. Assume the position of is given as a sinusoidal function of . Sketch a graph of the function, and then find a cosine function that gives the position in terms of .

